Northeastern University  Mathematics Department

Geometry, Physics, and Representation Theory Seminar

Allen Knutson

Cornell University

Thursday, January 16, 2:50-3:50 pm, Lake Hall 509

Grassmannians, puzzles, and quiver varieties

Abstract

Given four random red lines in 3-space, how many blue lines touch all four red? The answer is two, and this is the first nontrivial question in "Schubert calculus". Hilbert's 15th problem was to give this theory a solid foundation, which we now see as the cohomology ring of the Grassmannian of 1-planes in 3-space (or k-planes in affine n-space). There are many variations, all of which are easy to study algebraically, but only a few of which are understood combinatorially.

In the late '90s Terry Tao and I proved one could count "puzzles" in place of counting actual subspaces, and I solved similar problems with puzzles, some only conjecturally. In the last couple of years, through joint work with Paul Zinn-Justin, the geometry behind puzzles has become clearer: they are actually calculations on Nakajima quiver varieties (though for this talk I will mainly need spaces of diagonalizable complex matrices with fixed spectrum).