1) How many integers in the range 1 to 200 (inclusive) are not divisible by either 3 or 5?

Let \( A = \{x \in \mathbb{Z} : 1 \leq x \leq 200, \ 3 \mid x \} \), \( B = \{x \in \mathbb{Z} : 1 \leq x \leq 200, \ 5 \mid x \} \), \( U = \{x \in \mathbb{Z} : 1 \leq x \leq 200 \} \). Then the number in question is

\[
|U| - |A \cup B| = |U| - |A| - |B| + |A \cap B| = 200 - 66 - 40 + 13 = 107,
\]

(because \( A = \{3, 6, 9, \ldots, 198\} \), \( B = \{5, 10, 15, \ldots, 200\} \), \( A \cap B = \{15, 30, \ldots, 195\} \).

2) Let \( A, B \) and \( C \) denote sets. Prove or disprove: \((A - B) - C = A - (B \cup C)\).

First solution: \( x \in (A - B) - C \iff (x \in A) \land \neg (x \in B) \land \neg (x \in C) \equiv \neg (x \in B) \land (x \in A) \land \neg (x \in C) \equiv (x \in A) \land \neg (x \in B) \land \neg (x \in C) \).

From De Morgan's law, we see that two sets are equal.

Second solution: (by Venn diagrams):

\[
\begin{align*}
A \setminus B & \setminus C = A \setminus (B \cup C) \\
\text{same set.}
\end{align*}
\]

3) How many license plates are there consisting of two distinct digits and four distinct letters, if the digits and letters can follow in an arbitrary order?

\[
\frac{10 \cdot 9}{2} \cdot \frac{26 \cdot 25 \cdot 24 \cdot 23}{4 \cdot 3 \cdot 2} = \frac{6!}{2} \cdot \frac{10 \cdot 9 \cdot 26 \cdot 25 \cdot 24 \cdot 23}{2}.
\]

4) Extra credit. Let \( A \) be a set of cardinality \( n \geq 1 \). Prove that the set \( \{B \in 2^A : |B| \text{ is even} \} \) has cardinality \( 2^{n-1} \).

Encode subsets of \( A \) by length \( n \) bit strings. So the cardinality in question is the number of length \( n \) bit strings with an even number of 1's. Now each of the first \((n-1)\) bits can be chosen in one of two ways, but the last one is determined uniquely, giving the desired answer \( 2^{n-1} \).