The TVM Solver

Suppose you deposit $1000 in a bank account that pays 6% interest (APR) compounded monthly. Suppose you also deposit an additional $100 in the account at the end of every month. How do you figure out how much money you have in the bank at the end of 12 months?

One way is to construct a table.

<table>
<thead>
<tr>
<th>deposit</th>
<th>interest earned</th>
<th>balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>initial</td>
<td></td>
<td>$1,000</td>
</tr>
<tr>
<td>end of month 1</td>
<td>$100</td>
<td>$5.000000</td>
</tr>
<tr>
<td>end of month 2</td>
<td>$100</td>
<td>$5.525000</td>
</tr>
<tr>
<td>end of month 3</td>
<td>$100</td>
<td>$6.052625</td>
</tr>
<tr>
<td>end of month 4</td>
<td>$100</td>
<td>$6.582888</td>
</tr>
<tr>
<td>end of month 5</td>
<td>$100</td>
<td>$7.115803</td>
</tr>
<tr>
<td>end of month 6</td>
<td>$100</td>
<td>$7.651382</td>
</tr>
<tr>
<td>end of month 7</td>
<td>$100</td>
<td>$8.189638</td>
</tr>
<tr>
<td>end of month 8</td>
<td>$100</td>
<td>$8.730587</td>
</tr>
<tr>
<td>end of month 9</td>
<td>$100</td>
<td>$9.274240</td>
</tr>
<tr>
<td>end of month 10</td>
<td>$100</td>
<td>$9.820611</td>
</tr>
<tr>
<td>end of month 11</td>
<td>$100</td>
<td>$10.369714</td>
</tr>
<tr>
<td>end of month 12</td>
<td>$100</td>
<td>$10.921562</td>
</tr>
</tbody>
</table>

That is a lot of work.

The TVM Solver makes it easy. Access it by APPS/Finance/TVM Solver. Set N to 12, I to 6, PV to -1000, PMT to -100, P/Y to 12, C/Y to 12, and PMT to END. PV and PMT are negative because money coming out of your pocket is taken to be negative. These amounts are coming out of your pocket (and into the bank’s pocket). Now put the cursor on the FV line and press ALPHA-SOLVE. The calculator will determine that FV is 2295.234049. This is the money transfer that would leave you and the bank all even, neither owing money to the other. It is positive, because it goes into your pocket.

\[
\begin{array}{|c|c|}
\hline
N &= 12 \\
I\% &= 6 \\
PV &= -1000 \\
PMT &= -100 \\
FV &= \text{•} \\
P/Y &= 12 \\
\hline
\end{array}
\]

\[
\begin{array}{|c|c|}
\hline
N &= 12 \\
I\% &= 6 \\
PV &= -1000 \\
PMT &= -100 \\
FV &= 2295.234049 \\
P/Y &= 12 \\
\hline
\end{array}
\]

The initial FV entry is not shown above because it is irrelevant. The new FV value is computed using the values of N, I%, PV, and PMT; ignoring the the old FV value. The last two lines on the TVM display are not shown in order to save space. In all the problems we consider, C/Y (the number of times per year the interest is compounded) is always the same as P/Y (the number of times per year that payment is made), and payments are always made at the end of the payment period.
What is remarkable is that the TVM Solver calculates the value of any of the five variables N, I%, PV, PMT, and FV, if the other four values are known. We can, for example, answer any of the following questions:

1. Suppose you deposit $1000 in a bank account that pays 6% interest (APR) compounded monthly. Suppose you also deposit an additional $100 in the account at the end of every month. How long does it take for the balance to reach $4000?

2. Suppose you deposit an initial deposit in a bank account that pays 6% interest (APR) compounded monthly and that you also deposit an additional $100 in the account at the end of every month. How big does your initial deposit have to in order to ensure that you have $3000 at the end of 12 months?

3. Suppose you deposit $1000 in a bank account that pays 6% interest (APR) compounded monthly. Suppose you also make a fixed monthly deposit in the account at the end of every month. How big does your monthly deposit have to be in order to ensure that you have $3000 in the bank at the end of 12 months?

4. Suppose you deposit $1000 in a bank account that pays interest compounded monthly. Suppose you also deposit an additional $100 in the account at the end of every month. What interest rate will ensure that you have $2300 in the bank at the end of 12 months?

To answer the first question, for example, simply set FV to 4000, put the cursor on the N line, and press ALPHA-SOLVE. The calculator determines that N=26.77298872. We know that interest is paid only at the end of the month, so it will be 27 months before the balance goes over $4000. What is the balance after 27 months? Use the TVM Solver to figure it out. What is the balance after 26 months?

For practice, answer the other questions.

### Mortgage payments

Suppose you obtain a 30 year mortgage for $390,000 at 3.7%, compounded monthly. What are the monthly payments?

<table>
<thead>
<tr>
<th>N= 360</th>
<th>N= 360</th>
</tr>
</thead>
<tbody>
<tr>
<td>I%= 3.7</td>
<td>I%= 3.7</td>
</tr>
<tr>
<td>PV= 390000</td>
<td>PV= 390000</td>
</tr>
<tr>
<td>PMT=</td>
<td>PMT= -1795.10382</td>
</tr>
<tr>
<td>FV= 0</td>
<td>FV= 0</td>
</tr>
<tr>
<td>P/Y= 12</td>
<td>P/Y= 12</td>
</tr>
</tbody>
</table>

360 is 30 times 12, the number of payments. (You can enter this in the TVM Solver as 30*12 if want to.) PV is positive because that money goes into your pocket. PMT is negative because that money goes out of your pocket. FV is 0 because, at the end of 30 years, you and the bank are all even. This computation shows that if payments of $1,795.10382 are made each month, the amount still owed after 30 years will be exactly $0.
There is one subtle problem in determining the mortgage payments. You cannot make a payment in thousandths of a penny, which is what paying $1,795.10382 would require. Banks solve this problem by rounding the payment up (to $1,795.11 in this case) and making up for the slight monthly overpayment (for 29 years and 11 months) by reducing the final mortgage payment appropriately. What should the final payment be reduced to? This requires another TVM calculation. We compute FV using the actual mortgage payment.

\[
\begin{array}{c|c}
N= & 360 \\
I\%= & 3.7 \\
PV= & 390000 \\
PMT= & -1795.11 \\
FV= & 4.1853535 \\
P/Y= & 12
\end{array} \quad \xrightarrow{\text{ALPHA-SOLVE}} \quad \begin{array}{c|c}
N= & 360 \\
I\%= & 3.7 \\
PV= & 390000 \\
PMT= & -1795.11 \\
FV= & -248714.5385 \\
P/Y= & 12
\end{array}
\]

We conclude that after 30 years, the bank would end up owing $4.19 if payments of $1795.11 were made every month for 30 years. Instead, the final mortgage payment is reduced by $4.19 (to $1790.92).

The total amount paid to the bank over the 30 years is therefore

\[359 \times 1,795.11 + 1,790.92 = 646,235.41\]

Of this, $390,000 went to pay off the money borrowed and

\[646,235.41 - 390,000 = 256,235.41\]

was payed in interest charges.

The TVM Solver can also be used to answer other questions about the mortgage. Suppose, for example, you want to know what part of a mortgage payment goes to pay off the interest charges. This will vary over the life of the mortgage. Initially, most of the mortgage payment goes to pay interest charges because you owe so much money. At the end, very little of the mortgage payment goes to pay interest charges because you owe relatively little money at that point.

Suppose for example, you want to know how much the interest charges are halfway through the life of the mortgage. Specifically, suppose we want to know how much of the 180th payment goes to pay interest. We need to know how much we still owed to the bank after the previous payment, because that is the debt that we are paying interest on. So we need to know the balance due after the 179th payment. Using the TVM Solver:

\[
\begin{array}{c|c}
N= & 179 \\
I\%= & 3.7 \\
PV= & 390000 \\
PMT= & -1795.11 \\
FV= & \quad \text{ } \\
P/Y= & 12
\end{array} \quad \xrightarrow{\text{ALPHA-SOLVE}} \quad \begin{array}{c|c}
N= & 179 \\
I\%= & 3.7 \\
PV= & 390000 \\
PMT= & -1795.11 \\
FV= & -248714.5385 \\
P/Y= & 12
\end{array}
\]

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The interest payment is obtained by multiplying the value of FV by \( .037/12 \) (1/12 of 3.7% of the value of FV). You might be tempted to copy 248714.5385 to a piece of paper, then copy it back to the main screen so that you can multiply it by \( .037/12 \). *Don’t do this.* Learn to operate your calculator properly. Your calculator stores the results it obtains (or has the ability to store them if you so request) and you should learn how to retrieve these results for use in further computations. The results of a TVM computation are stored in the variables \( \text{tvm}_\text{Pmt}, \text{tvm}_\text{I%}, \text{tvm}_\text{PV}, \text{tvm}_\text{N}, \) and \( \text{tvm}_\text{FV} \). So you should compute the interest payment in the 180th payment to the bank by evaluating \( \text{tvm}_\text{FV}.037/12 \) on your main screen. You get \( \text{tvm}_\text{FV} \) on your main screen via the APPS/Finance menu. The result is \(-766.8698272\).

**Mortgage tables; a worked example**

Example problem: You borrow $250,000 at 6.1% compounded monthly for 30 years.

a. What are your monthly payments?

b. Fill in the interest and balance in the blanks below:

<table>
<thead>
<tr>
<th>Interest charge</th>
<th>Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 month A1</td>
<td>B1</td>
</tr>
<tr>
<td>2 months A2</td>
<td>B2</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>8 yrs, 11 months</td>
<td>B3</td>
</tr>
<tr>
<td>9 yrs A3</td>
<td>B4</td>
</tr>
<tr>
<td>...</td>
<td>...</td>
</tr>
<tr>
<td>29 yrs, 11 months</td>
<td>B5</td>
</tr>
<tr>
<td>30 yrs A4</td>
<td>B6</td>
</tr>
</tbody>
</table>

(In order to make the explanation which is given below of how to fill in the table clear, the blanks are labeled A1, ..., A4 and B1, ..., B6.)

c. What is the final payment?

d. What is the total interest paid over the life of the loan?

a. The TVM Solver gives \(-1514.9869\ldots\) for PMT. The bank always rounds this up, so the payment is $1,514.99 per month.

b. A1: Evaluate \( .061/12*250000 \).

Set PMT to 1514.99 (the actual interest payment) in the TVM Solver.
B1: Set \( N \) to 1 and solve for \( FV \). The result is \(-249,755.8433\). This says that after one month, you owe the bank $249,755.84.

A2: Evaluate \( 0.061/12 \times tvm_{-}FV \).

B2: Set \( N \) to 2 and solve for \( FV \).

B3: Set \( N \) to \( 8 \times 12 + 11 \) and solve for \( FV \).

A3: Evaluate \( 0.061/12 \times tvm_{-}FV \).

B4: Set \( N \) to \( 9 \times 12 \) and solve for \( FV \).

B5: Set \( N \) to \( 29 \times 12 + 11 \) and solve for \( FV \).

A4: Evaluate \( 0.061/12 \times tvm_{-}FV \).

B6: This is \( 0 \).

The relevant rows of the completed table are:

<table>
<thead>
<tr>
<th>Interest charge</th>
<th>Balance</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 month</td>
<td>$1,270.33</td>
</tr>
<tr>
<td>2 months</td>
<td>$1,269.59</td>
</tr>
<tr>
<td>8 yrs, 11 months</td>
<td>$215,400.08</td>
</tr>
<tr>
<td>9 yrs</td>
<td>$1,094.95</td>
</tr>
<tr>
<td>29 yrs, 11 months</td>
<td>$1,504.23</td>
</tr>
<tr>
<td>30 yrs</td>
<td>$7.65</td>
</tr>
</tbody>
</table>

C. Set \( N=30 \times 12 \) and solve for \( FV \). The result is \( 3.108838 \). This says that if the monthly payment of \( 1514.99 \) were continued for the full 30 years, the bank would end up \( owing you \$3.11 \). This is not what is done. Instead, the final monthly payment is reduced by \$3.11, to \$1,511.88, so that the final balance is exactly \$0.

d. The total amount of money that is paid to the bank is \( 359 \times \$1514.99 + \$1511.88 = \$545,393.29 \) (359 regular payments and one final reduced payment). The total interest payments are therefore \$545,393.29 - \$250,000 = \$295,393.29.