Quadric Surfaces in Maple

0.1 Maple and Implicit Plots

Maple is very powerful mathematics manipulation and plotting software. It can do symbolic algebra and calculus, compute derivatives and indefinite integrals, and perform complicated numerical calculations. We will not use its full power here, but simply draw upon certain of its plotting abilities not available in Matlab.

As we have seen above, if we are given a function \( f(x, y) \) of two variables, we can write \( z = f(x, y) \) and plot the graph of \( f \), i.e. the set of points \((x, y, f(x, y))\). This is called an explicit plot since the variable \( z \) is given explicitly as a function of the variables \( x \) and \( y \). A more general example of an explicit plot is a parametric plot in which each of the variables \( x, y, \) and \( z \) is given explicitly as a function of one or more other variables called parameters. For example, the helix curve given by \( x = 3 \cos(t), y = 3 \sin(t), z = 2t \) or the sphere given by \( x = \sin(s) \cos(t), y = \sin(s) \sin(t), z = \cos(s) \) are explicit plots.

The general equation for a quadric surface, however, is:

\[
Ax^2 + By^2 + Cz^2 + Dxy + Eyz + Fzx + Gx + Hy + Iz + J = 0.
\]

No variable here is given explicitly in terms of the others. Although it is possible to solve for \( z \), for example, in terms of the other variables, it involves the quadratic formula and usually requires two separate expressions corresponding to the two roots; in other words, it is pretty messy and awkward to work with. The surface which consists of all points \((x, y, z)\) satisfying this equation is said to be defined implicitly (i.e. there is an implied solution of one variable in terms of the others, but it is not given explicitly).

Unlike Matlab, Maple provides a plotting procedure which allows you to draw surfaces defined implicitly.

0.2 A Maple Implicit Plot Example

Call up the Maple program using NUNET Programs from the Start menu (go to Computational and Statistical packages, then choose Maple). The Maple prompt looks like this: \( > \). Here is a sample Maple “session”, followed by an explanation. You should try typing this in for practice. Be very careful in entering the commands. Maple is “case-sensitive” which means that it distinguishes between capital and lower-case letters (for example, “Pi” denotes the numerical constant 3.14159..., while “pi” is just the greek letter \( \pi \); and, of course, correct spelling is important.
This plot produces a hyperboloid of 1-sheet, a vertical cylinder, and a plane (with x missing), all on the same set of axes.

```plaintext
> with(plots):
> f := x^2+y^2-z^2 = 9;
> g := x^2+y^2 = 4;
> h := z = y;
> implicitplot3d({f, g, h}, x=-5..5, y=-5..5, z=-7..7, axes=boxed, grid=[12,12,12], style=patchcontour);
```

Here are some additional points about Maple in general and the specific session above.

- Every command in Maple, in order to be executed, must end with a colon (:) or semicolon (;). In general, the semicolon stops Maple from printing any response to the command; thus, the first line above has the command “with(plots);”. This loads Maple’s graphing package. Had there been a semicolon, Maple would have printed out the names of the many subroutines in these packages — obviously something not very useful.

- Each time you start Maple, and want to draw a plot, you have to execute the “with(plots);” command; however, you only have to execute it once for each Maple session.
• If you have created the plot above, and want to change one of the functions, use the mouse to position the cursor appropriately on the function, and change it. In order for this change to register with Maple, you must press Enter with the cursor on this function line. This is true in general for making changes in Maple commands.

• To insert a new command line before an already typed Maple line, use the Insert Menu at the top.

• You can change various attributes of the plot (e.g. type of axes, colors, etc.). Click anywhere in the plot; this selects the plot with a box, and adds new menu icons at the top of the screen. For example, from these icons, choose Style then Patch w/o grid (instead of the patchcontour typed in above); choose Color then Z(Grayscale) (best for printing) or Z(Hue) (good for viewing). Play around with these (e.g. some of the Projection choices — this plot uses the default far).

• Once the plot is on the screen, you can rotate it in 3-space by dragging it with the mouse: experiment with this — use small changes.

• Observe carefully how the equations \( f, g, h \) were entered in the sample lines above. Note that they are put in braces: \{f,g,h\} in the implicitplot command.

• You can use the Help menu at the top to learn more about Maple; also, ask the mentor(s) in the Math Dept. Computer Lab (in 553 Lake).

0.3 Quadric Surfaces Exercises to Hand In

Any of the quadric surfaces (Hyperboloids, Paraboloids, Ellipsoids, Cones and Cylinders based on parabolas, ellipses or hyperbolas) can be rotated in such a way that its equation does not have any \( xy, yz, \) or \( zx \) terms. Equivalently, we can let the coefficients of these terms (\( D, E, F \)) in the general equation above be equal to 0, and still get any kind of quadric surface. However, sometimes you encounter a second-degree equation having these terms, and you have to identify it. The simplest way is to plot it.

Use Maple and implicitplot3d to identify the surfaces given by the equations below. In each case attach a printout of the plot, and give the name of the surface (for example, “Hyperboloid of 1 sheet”). At least for starters, include the following commands \( x=-3..3, y=-3..3, z=-3..3, \) axes=boxed, grid=[12,12,12]. (NOTE: You only want to plot one surface at a time, so modify the example above by deleting the \( g \) and \( h \) from the implicitplot3d command, and use only \( f \).) Once you get a plot, you may want to change some or all of these. Remember that you can also rotate the plot to get a better viewpoint: click on it once so that a frame forms around it (this will also give various
plotting options on the menu bar at the top of the screen); now dragging the plot in various directions will cause it to rotate. Do this slowly and carefully; it takes a while to get used to how it works. Remember that certain surfaces, from certain viewpoints, may look the same. For example, the bottom half of an ellipsoid can look like the top part of a hyperboloid of two sheets or like a paraboloid. Try to rotate your plot so that you can be pretty sure which surface you’re dealing with.

a) \(2xy - z^2 + yz - x = 1\)

b) \(2x^2 + x - z - xy + 2yz = 1\)

c) \(xz + 3yz = 1\)

d) \(2x^2 + 3y^2 + 4z^2 - 3xy + yz = 8\)

e) \(2x^2 + 3y^2 + 4z - xy + yz - 4y = 1\)

f) \(xy - 3yz + x = 1\)

Finally, you can probably fit at least two of these plots on a page: this will save time and paper!