Instructions: Put your name and your instructor’s name in the blanks above. Put your final answers to each question in the designated spaces on these test pages. **SHOW ALL WORK** (e.g. integrals from a calculator without your work are worth 0). If there is not enough room to show your work, use the back of the preceding page. Show all calculator computations to at least four significant digits.

1. Evaluate the following integrals (3 pts each).

   (a) \[ \int \frac{\arcsin(x)}{\sqrt{1 - x^2}} \, dx \]

   (b) \[ \int x \sqrt{x^2 - 9} \, dx \]

   (c) \[ \int \frac{\ln(z)}{z^3} \, dz \]

   (d) \[ \int te^{5t} \, dt. \]
2. Solve the initial-value problem \( \frac{dy}{dx} = (5 \sin x + 4)e^{y} \), where \( y = 0 \) when \( x = \frac{\pi}{2} \) (6 pts).

3. A force \( \mathbf{F} = (1, -4, 4) \) (in newtons) moves an object on a straight line from \( \mathbf{P} = (-1, 2, 8) \) to \( \mathbf{Q} = (1, 5, 10) \).

   (a) Find the displacement vector \( \overrightarrow{PQ} \) (2 pts).

   (b) Find the angle between \( \mathbf{F} \) and \( \overrightarrow{PQ} \) (in radians) (3 pts).

   (c) Find the vector projection of \( \mathbf{F} \) on \( \overrightarrow{PQ} \) (i.e. \( \text{Proj}_{\overrightarrow{PQ}} \mathbf{F} \)) (2 pts).

   (d) Find the work done by \( \mathbf{F} \) acting through \( \overrightarrow{PQ} \) (3 pts).
4. Beginning at time $t = -5$ and ending at time $t = 5$ (in seconds), a particle moves along a curve $C$ so that its coordinates (in meters) at time $t$ are given by: 
\[
\begin{align*}
x(t) &= t^3 - 12t \\
y(t) &= t^2 + 1
\end{align*}
\]. Here is a plot:

(a) Find the velocity vector $\mathbf{v}(t)$ at time $t$ and the speed at $t = 2$ (3 pts).

(b) Find the acceleration vector $\mathbf{a}(t)$ at time $t$ (2 pts).

(c) Draw an arrow in the above plot to indicate the direction of motion (increasing $t$) (2 pts).

(d) Find the times $t$ at which the velocity vector is vertical, and the times at which it is horizontal. Find the $x$ and $y$ coordinates of these points (5 pts).

(e) Set up but don’t evaluate, an integral to calculate the total distance traveled by the particle during the time interval (3 pts).

5. Determine if the following improper integrals converge or diverge, and compute their value if they converge (3 pts each).

(a) \[
\int_{-3}^{3} \frac{4}{(x-1)^{3}} \, dx
\]

(b) \[
\int_{0}^{1} x^{-0.2} \, dx
\]

(c) \[
\int_{0}^{\infty} 2x^3 e^{-x^2} \, dx \quad \text{(HINT: } \int 2x^3 e^{-x^2} \, dx = -x^2 e^{-x^2} - e^{-x^2} \text{ and } \lim_{x \to \infty} x^n e^{-x^n} = 0.)\]


6. Let \( R \) be the region bounded above by \( y = 6 - x^2 \) and below by \( y = 2 \).
   
   (a) Find the volume obtained when \( R \) is rotated around the \( x \)-axis (6 pts).

   (b) Set up, but don’t evaluate, an integral to find the volume obtained when \( R \) is rotated around the line \( y = -3 \) (4 pts).

7. A tank is shaped like an inverted cone. Its radius is 4 feet and its height is 8 feet. Its base is 3 feet below ground, and it is filled with water. A cubic foot of water weighs about 62.428 pounds and, for a cone, Volume = \( \frac{1}{3} \pi R^2 H \). Set up completely, but don’t evaluate, an integral to calculate how much work is done in pumping the contents of the tank to ground level (8 pts).
8. Set up integral(s) to calculate the area of the region shown below. Do not evaluate these integrals (7 pts).

![Graph showing the region bounded by y = \sqrt{x}, y = x - 6, and y = -x.]

9. Determine which of the following series converge or diverge. In each case, describe the test you used (3 pts each).

(a) \( \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2 + 3n} \)

(b) \( \sum_{n=1}^{\infty} \frac{n^2}{n^2 + 3n} \)

(c) \( \sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + 1} \)
10. Find the Taylor series for \( f(x) = \cos x \), centered at \( \pi \) (5 pts).

11. \( \ln \left( \frac{x}{3} \right) = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}(x-3)^k}{k \cdot 3^k} \)

(a) Find the radius of convergence for this series (4 pts).

(b) Find an interval of convergence for this series—don’t bother to check endpoints (2 pts).

(c) Use 4 terms of this series to approximate \( \ln(4/3) \), and estimate the error using the fact that this series alternates (3 pts).