Quiz 2.1  MTHU241  Fall 2004  Name:  Answers

Please give your answers clearly in the space provided on this sheet. Show work where appropriate, possibly on the back.  Point total = 20

(1) (6 points) Find and simplify a formula for the average rate of change of the function \( f(x) = \frac{1}{5x - 7} \) from \( x = c \) to \( x = c + h \). Show your work in complete detail. Your work is your answer.

\[
\frac{f(c + h) - f(c)}{h} = \frac{1}{5(c + h) - 7} - \frac{1}{5c - 7} = \frac{1}{h(5c + 5h - 7)(5c - 7)} \cdot \frac{5c - 7 - (5c + 5h - 7)}{h(5c + 5h - 7)(5c - 7)} = \frac{-5h}{h(5c + 5h - 7)(5c - 7)} = \frac{-5}{(5c + 5h - 7)(5c - 7)}
\]

(2) (4 points) USING your ANSWER to PROBLEM 1, find \( f'(c) \), where, as in problem 1, \( f(x) = \frac{1}{5x - 7} \). (This takes little additional work, but correct use of notation is important. Make sure you show how you use the previous result.)

\[
f'(c) = \lim_{h \to 0} \frac{f(c + h) - f(c)}{h}. \text{ By the previous result, this is } \lim_{h \to 0} \frac{-5}{(5c + 5h - 7)(5c - 7)} = \frac{-5}{(5c - 7)^2}.
\]

[Note: Proper use of the limit notation is essential. Failure to use it at all will cost all 4 points.]

(3) (5 points) Use (“shortcut”) rules of differentiation to find the following derivatives:

(a) \( \frac{d}{dx} (3x^7 + 5x - \pi^2) = 21x^6 + 5 \).

(b) \( \frac{dt^{1/3}}{dt} = (1/3)t^{-2/3} \).

(4) (5 points) Find the equation of the tangent line to the graph of \( y = x^3 \) at \( x = 5 \). Show work.

The equation is: \( y = 75(x - 5) + 125 \).

\( y'(5) = 3(5^2) = 75 \) is the slope, and the line passes through the point \( (5, 5^3) = (5, 125) \), so by the point-slope formula the equation of the tangent line is as shown. [The point-slope formula for a line of slope \( m \) passing through \( (x_1, y_1) \) is \( y = m(x - x_1) + y_1. \) It is the most useful form of a straight line equation in many cases.]
Quiz 2.2  MTHU241  Fall 2004  

Please give your answers clearly in the space provided on this sheet. Show work where appropriate, possibly on the back.  

Point total = 20

(1) (6 points) Find and simplify a formula for the average rate of change of the function \( f(x) = \frac{1}{7x - 5} \) from \( x = b \) to \( x = b + h \). Show your work in complete detail. Your work is your answer.

\[
\begin{align*}
\frac{f(b + h) - f(b)}{h} &= \frac{1}{7(b + h) - 5} - \frac{1}{7b - 5} \\
&= \frac{7b - 5 - (7b + 7h - 5)}{h(7b + 7h - 5)(7b - 5)} \\
&= \frac{-7h}{h(7b + 7h - 5)(7b - 5)} \\
&= \frac{-7}{(7b + 7h - 5)(7b - 5)}
\end{align*}
\]

(2) (4 points) **USING your ANSWER to PROBLEM 1,** find \( f'(b) \), where, as in problem 1, \( f(x) = \frac{1}{7x - 5} \). (This takes little additional work, but correct use of notation is important. Make sure you show how you use the previous result.)

\[
f'(b) = \lim_{h \to 0} \frac{f(b + h) - f(b)}{h}. \]

By the previous result, this is

\[
\lim_{h \to 0} \frac{-7}{(7b + 7h - 5)(7b - 5)} = \frac{-7}{(7b - 5)^2}.
\]

[Note: Proper use of the limit notation is essential. Failure to use it at all will cost all 4 points.]

(3) (5 points) Use (“shortcut”) rules of differentiation to find the following derivatives:

(a) \( \frac{d}{dx} (7x^4 - 9x + \pi^3) = 28x^3 - 9 \)

(b) \( \frac{du^{1/5}}{du} = (1/5)u^{-4/5} \)

(4) (5 points) Find the equation of the tangent line to the graph of \( y = x^5 \) at \( x = 2 \). Show work.

The equation is: \( y = 80(x - 2) + 32 \)

\( y'(2) = 5(2^4) = 80 \) is the slope, and the line passes through the point \((2, 2^5) = (2, 32)\), so by the point-slope formula the equation of the tangent line is as shown.  

[The point-slope formula for a line of slope \( m \) passing through \((x_1, y_1)\) is \( y = m(x - x_1) + y_1 \). It is the most useful form of a straight line equation in many cases.]