
**Problem 1.** Suppose \( V \) is to be found from the formula \( V = \frac{T}{P+T} \), where \( T \) and \( P \) are found to be 9 and 1 with maximal possible error \( |dT| = 0.5 \) and \( dP = 0.1 \). Estimate the maximum possible error in the computed value of \( V \).

**Solution.**

\[
V_T(T,P) = \frac{1}{(P + T) - T/(P + T)^2} \implies V_T(9,1) = 0.01.
\]

\[
V_P(T,P) = -T/(P + T)^2 \implies V_P(9,1) = -0.09.
\]

\[
dV = V_T(9,1)dT + V_P(9,1)dP \leq |V_T(9,1)|dT + |V_P(9,1)|dP \leq 0.01 \cdot 0.5 + 0.09 \cdot 0.1 = 0.014.
\]

**Answer:** The maximal possible error is 0.014.

**Problem 3.** If \( r = 10.0 \text{cm} \) and \( h = 8.0 \text{cm} \) to the nearest millimeter, what should we expect the maximum percentage error in calculating \( V = \pi r^2 h \)?

**Solution.**

\[
\begin{align*}
V_r(r,h) &= 2\pi rh, \\
V_h(r,h) &= \pi r^2.
\end{align*}
\]

\[
dV = V_r dr + V_h dh \implies \frac{dV}{V} = \frac{V_r}{V} dr + \frac{V_h}{P} dP = \frac{2dr}{r} + \frac{dh}{h} \leq \frac{2 \cdot 0.1}{10} + \frac{0.1}{8} \approx 0.03.
\]

\[
dV \cdot 100\% \leq 3\%.
\]

**Answer:** The maximal percentage error is 3%.

**Problem 5.** Find and classify the critical points of the function

(a) \( f(x,y) = xy(x+y-1) \);

**Solution.**

\[
\begin{align*}
f_x &= y(x+y-1)+xy = 0 \implies y = 0 \text{ or } x+y-1+x = 2x+y-1 = 0. \\
f_y &= x(x+y-1)+xy = 0 \implies x = 0 \text{ or } x+y-1+y = x+2y-1 = 0.
\end{align*}
\]

If \( y = 0 \) then the second equation implies \( x = 0 \). If \( x = 0 \) the first equation implies \( y = 0 \). Thus we have one critical point \((0,0)\). The other critical points must satisfy

\[
2x+y-1 = 0 \quad \text{and} \quad x+2y-1 = 0.
\]

Solving this 2 equations we obtain \( x = 1/3, \ y = 1/3 \).

Thus there are 2 critical points \((0,0)\) and \((1/3,1/3)\).

\[
\begin{align*}
f_{xx} &= 2y, \\
f_{yy} &= 2x, \\
f_{xy} &= f_{yx} = 2x+2y-1.
\end{align*}
\]

\[
(0,0) \quad f_{xx}(0,0) = 0, \ f_{yy}(0,0) = 0, \ f_{xy}(0,0) = -1. \quad \text{Thus} \\
D(0,0) = f_{xx}(0,0)f_{yy}(0,0) - f_{xy}(0,0)^2 = -1 < 0 \implies (0,0) \text{ is a saddle point.}
\]

\[
(1/3,1/3) \quad f_{xx}(1/3,1/3) = 2/3, \ f_{yy}(1/3,1/3) = 2/3, \ f_{xy}(1/3,1/3) = 1/3. \quad \text{Thus} \\
D(0,0) = f_{xx}(0,0)f_{yy}(0,0) - f_{xy}(0,0)^2 = 1/3 > 0; \ f_{xx}(1/3,1/3) = 2/3 > 0 \implies (1/3,1/3) \text{ is a minimum.}
\]

**Answer:** The function has a saddle point at \((0,0)\) and a minimum at \((1/3,1/3)\).